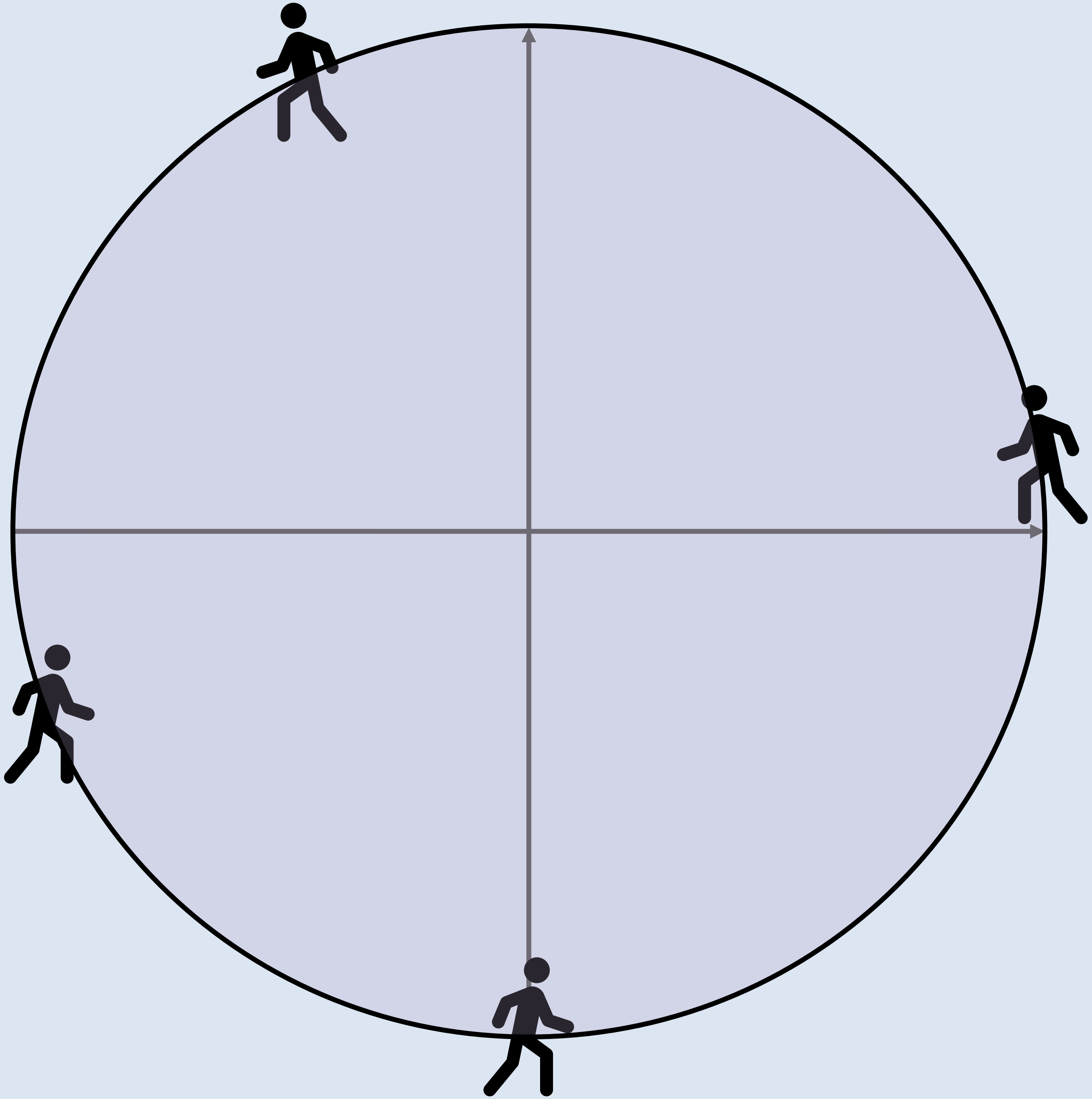
FOOTING

Romane FERRANDIS et Lucie CORREIA

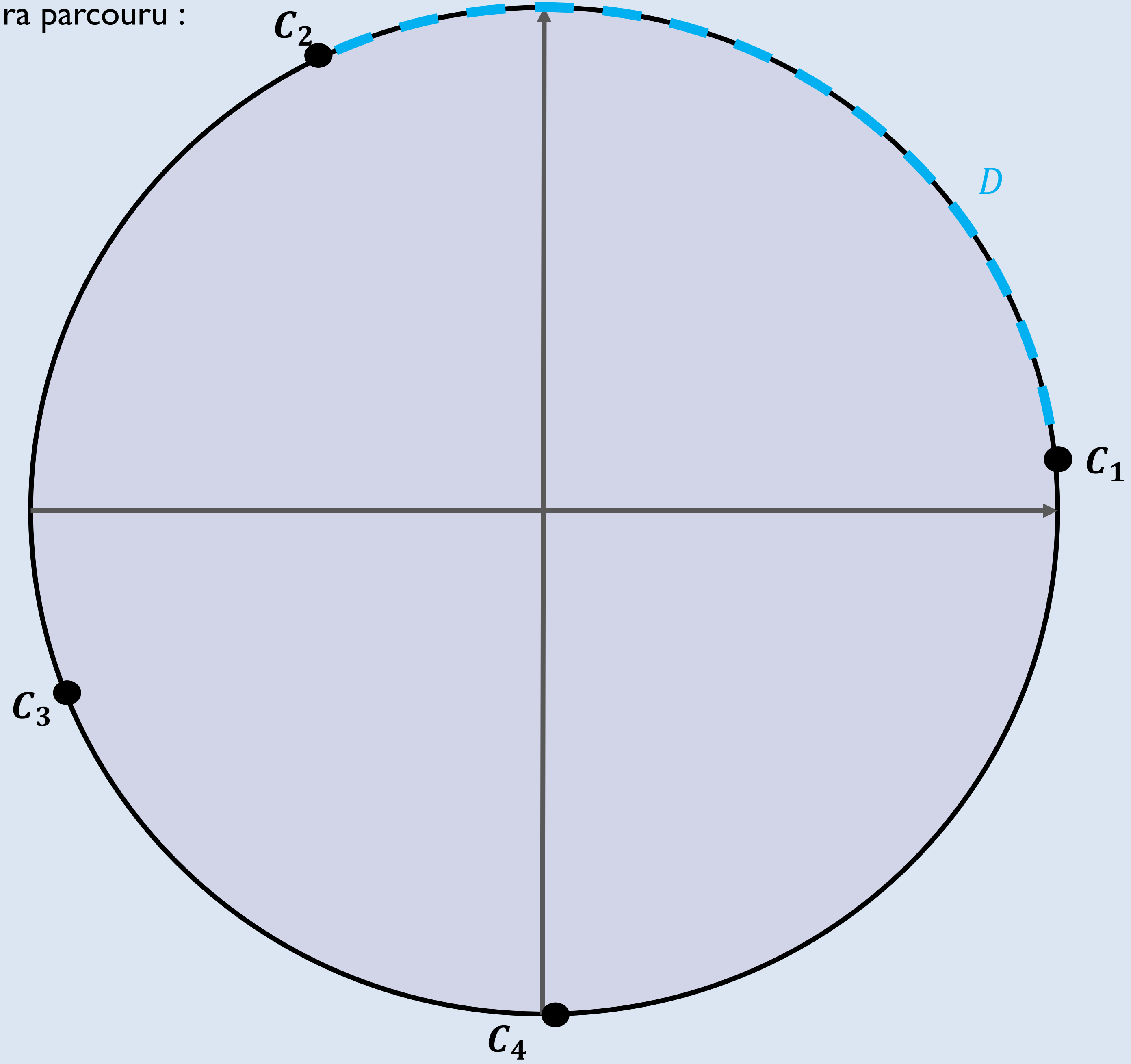


Si à l'instant n, D sépare $\boldsymbol{C_1}$ de $\boldsymbol{C_2}$, alors à l'instant $n+1$, $\boldsymbol{C_1}$ aura parcouru :

$$\boxed{\boldsymbol{d = p \times D}}$$

Avec $\boldsymbol{p} \in [\mathbf{0}; \mathbf{1}]$ fixé

Un exemple $\boldsymbol{p = 0,5}$



Si à l'instant n, D sépare C_1 de C_2, alors à l'instant $n+1$, C_1 aura parcouru :

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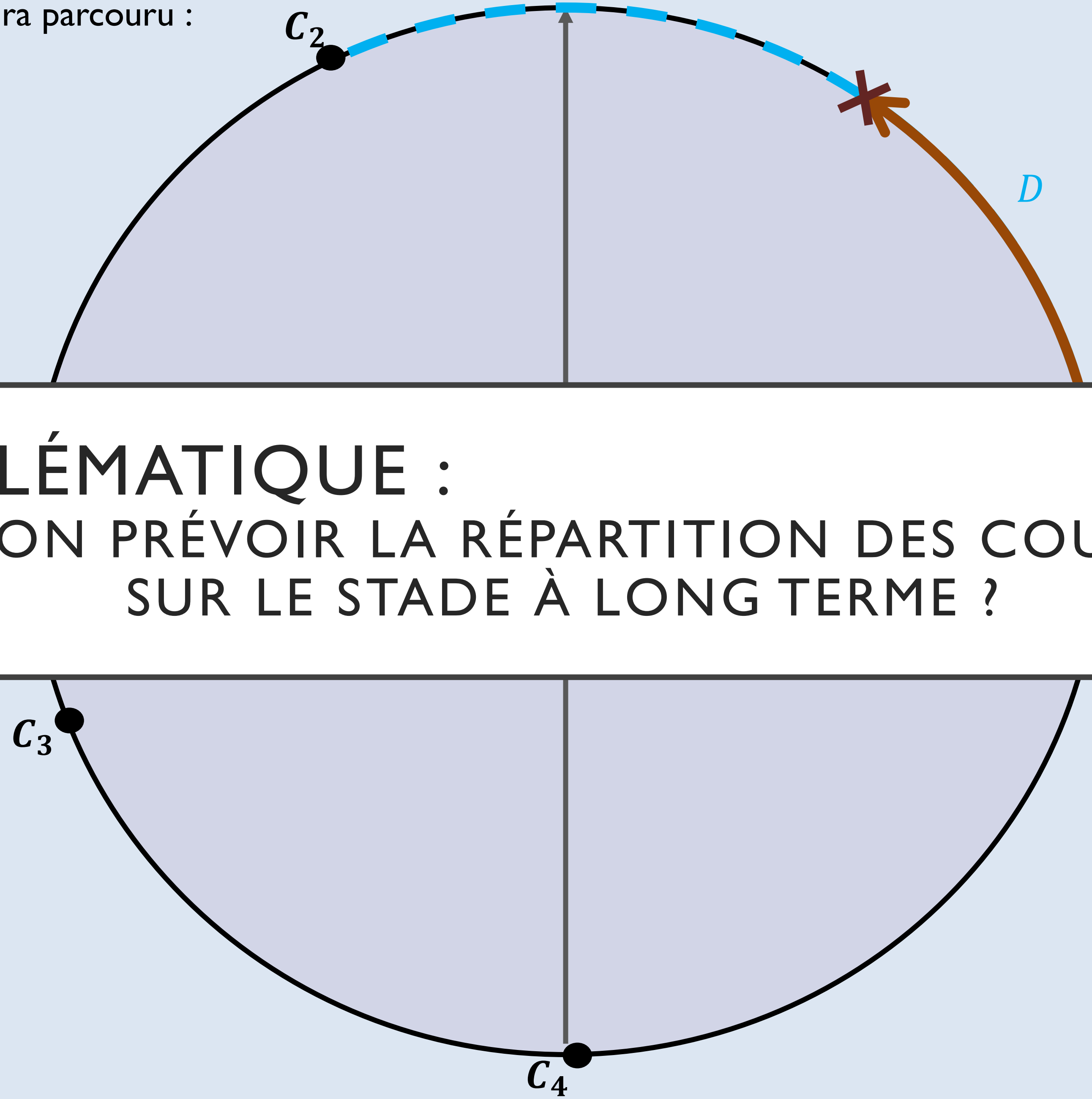
Un exemple $p = 0,5$



PROBLÉMATIQUE :

PEUT-ON PRÉVOIR LA RÉPARTITION DES COUREURS SUR LE STADE À LONG TERME ?

Si à l'instant n, D sépare $\boldsymbol{C_1}$ de $\boldsymbol{C_2}$, alors à l'instant $n+1$, $\boldsymbol{C_1}$ aura parcouru :

$$\boxed{\boldsymbol{d = p \times D}}$$

Avec $\boldsymbol{p} \in [\boldsymbol{0}; \boldsymbol{1}]$ fixé

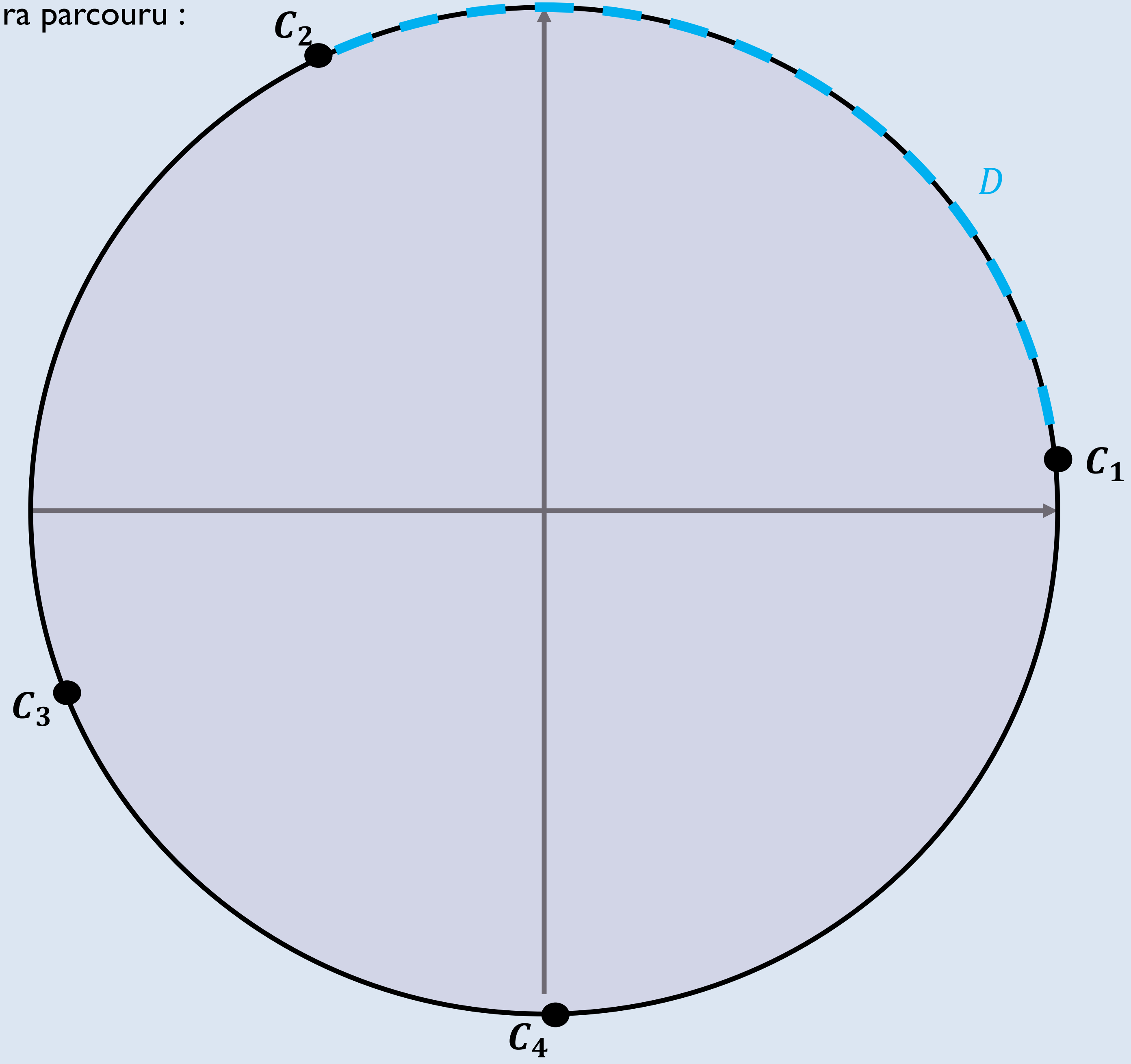
Cas Particuliers :

$\boldsymbol{p = 0}$

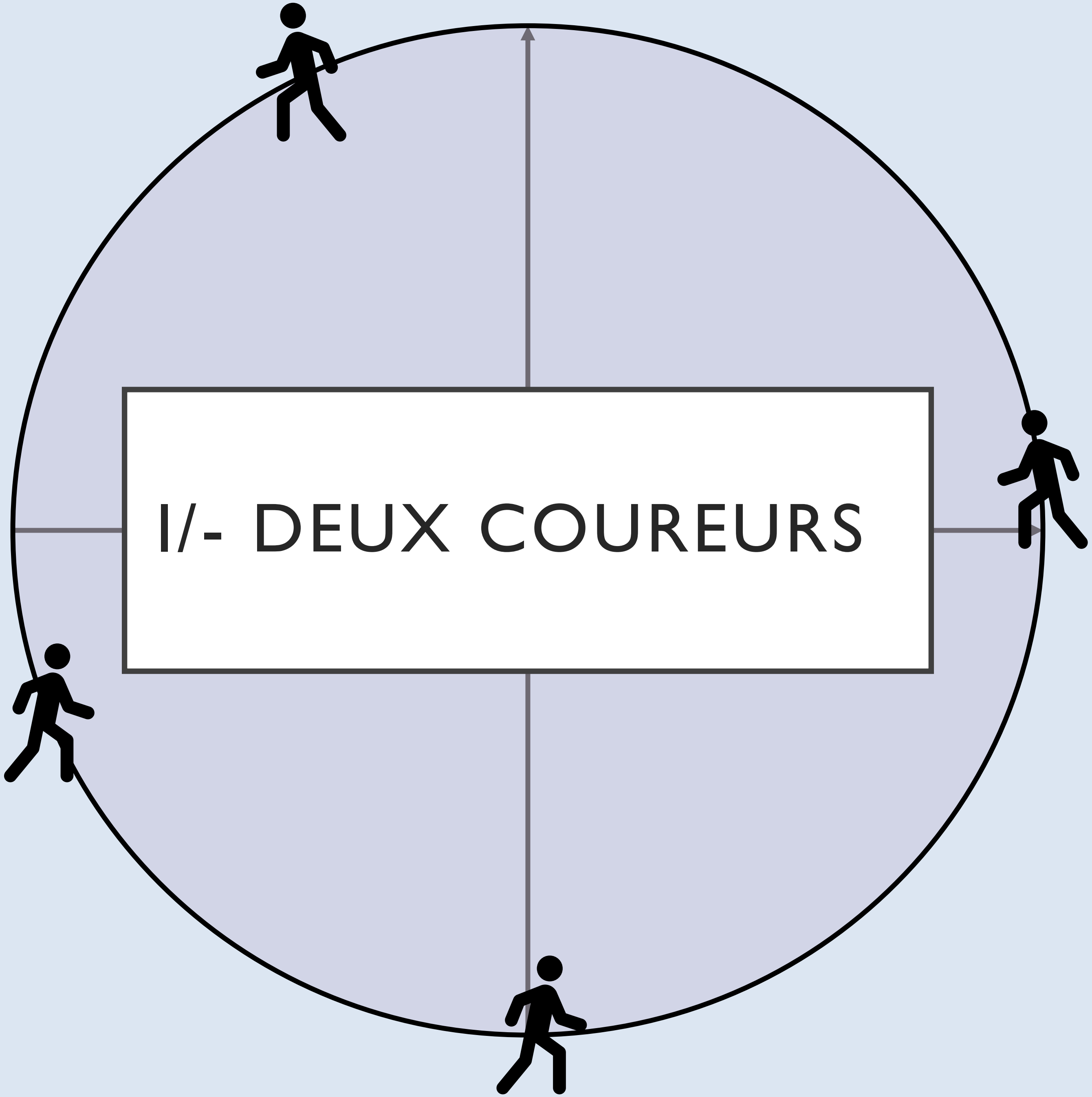
$\boldsymbol{p = 1}$

Donc, on établit :

$\boldsymbol{p} \in]\boldsymbol{0}; \boldsymbol{1}[$



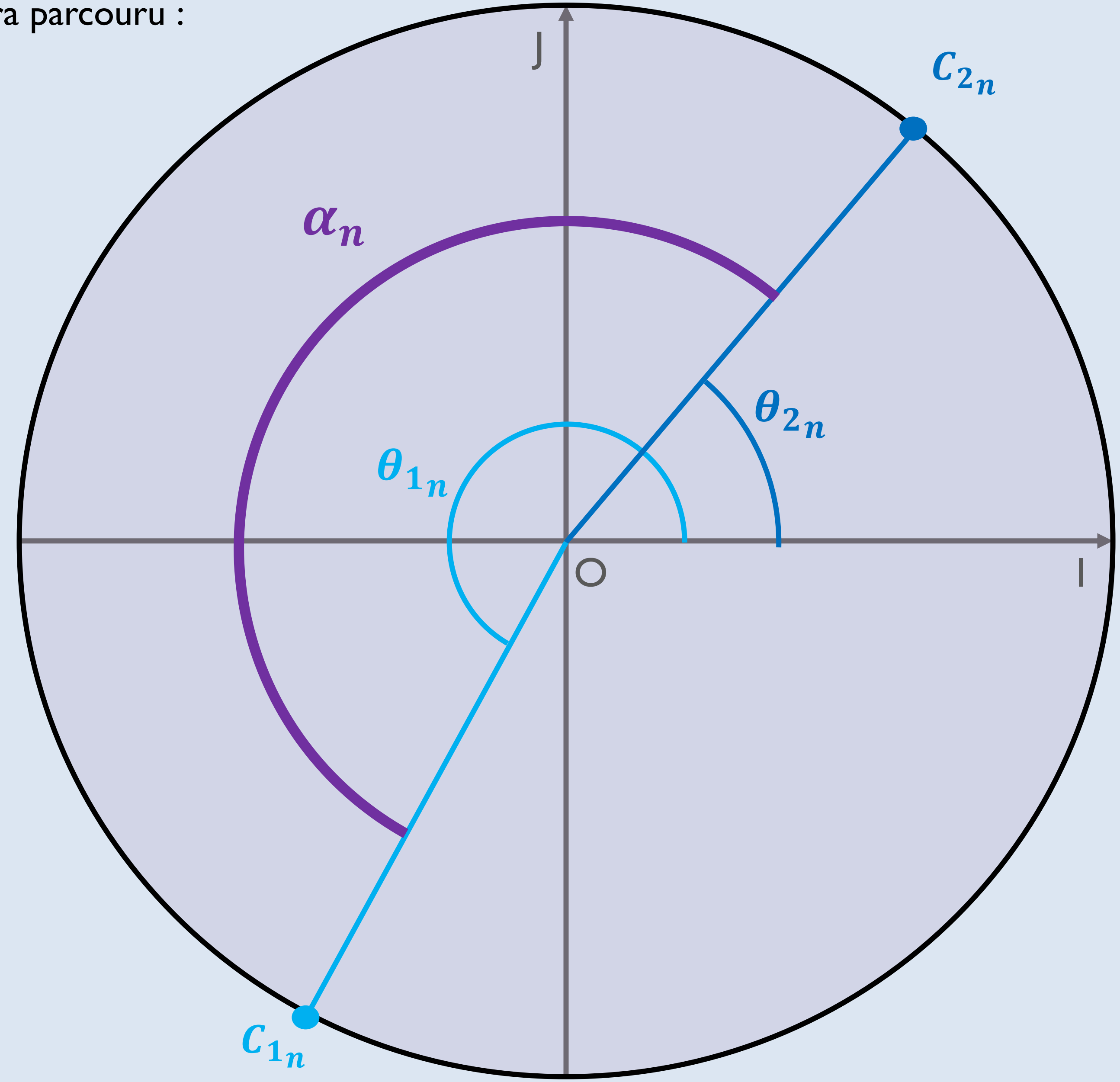
I/- DEUX COUREURS



Si à l'instant n, D sépare $\boldsymbol{C_1}$ de $\boldsymbol{C_2}$, alors
à l'instant $n+1$, $\boldsymbol{C_1}$ aura parcouru :

$$\boxed{\boldsymbol{d = p \times D}}$$

Avec $\boldsymbol{p} \in \left]\mathbf{0};\mathbf{1}\right[$ fixé



$$\boxed{\boldsymbol{\alpha_n = \theta_{1_n} - \theta_{2_n}}}$$

On obtient par le calcul une situation de récurrence :

$$\alpha_{n+1} = \theta_{1_{n+1}} - \theta_{2_{n+1}}$$

$$\boxed{\alpha_{n+1} = (-2p + 1)\alpha_n + 2p\pi}$$

Si (α_n) admet un limite l, alors :

$$\boxed{l = (-2p + 1)l + 2p\pi}$$

$$\boxed{l = \lim_{n\to\infty} \alpha_n = \pi}$$

$$\boxed{\alpha_n = \theta_{1_n} - \theta_{2_n}}$$

$$\boxed{l \overset{?}{=} \lim_{n\to\infty} \alpha_n \overset{?}{=} \pi}$$

LA SUITE CONVERGE-T-ELLE VRAIMENT ?

$$\boxed{u_n = \alpha_n - \pi}$$

$$\boxed{u_{n+1} = (-2p+1)u_n}$$

(u_n) géométrique de raison $-2p+1 \in]-1;1[$

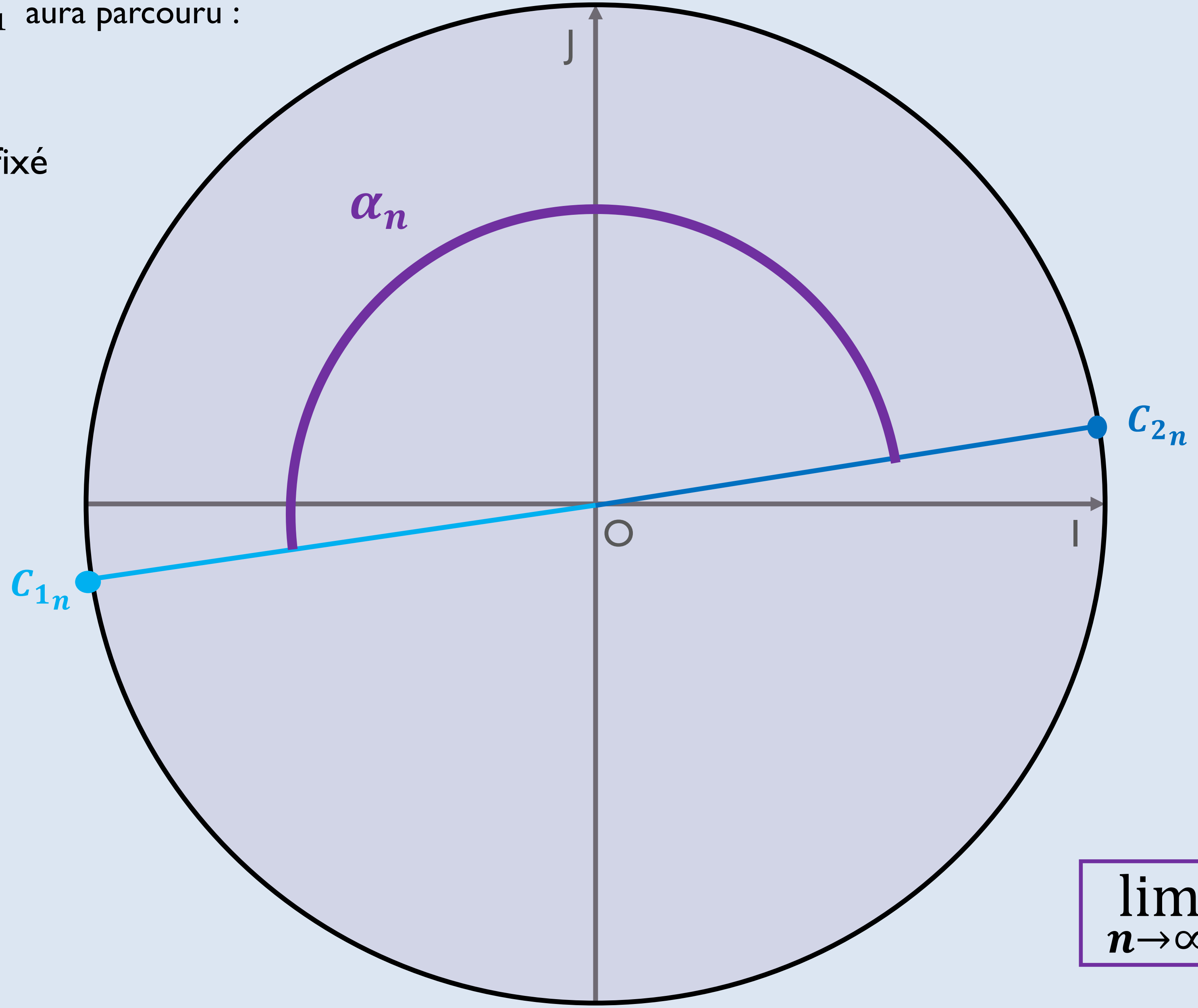
$$\boxed{\lim_{n\to\infty} u_n = 0}$$

$$\boxed{\lim_{n\to\infty} \alpha_n = \pi}$$

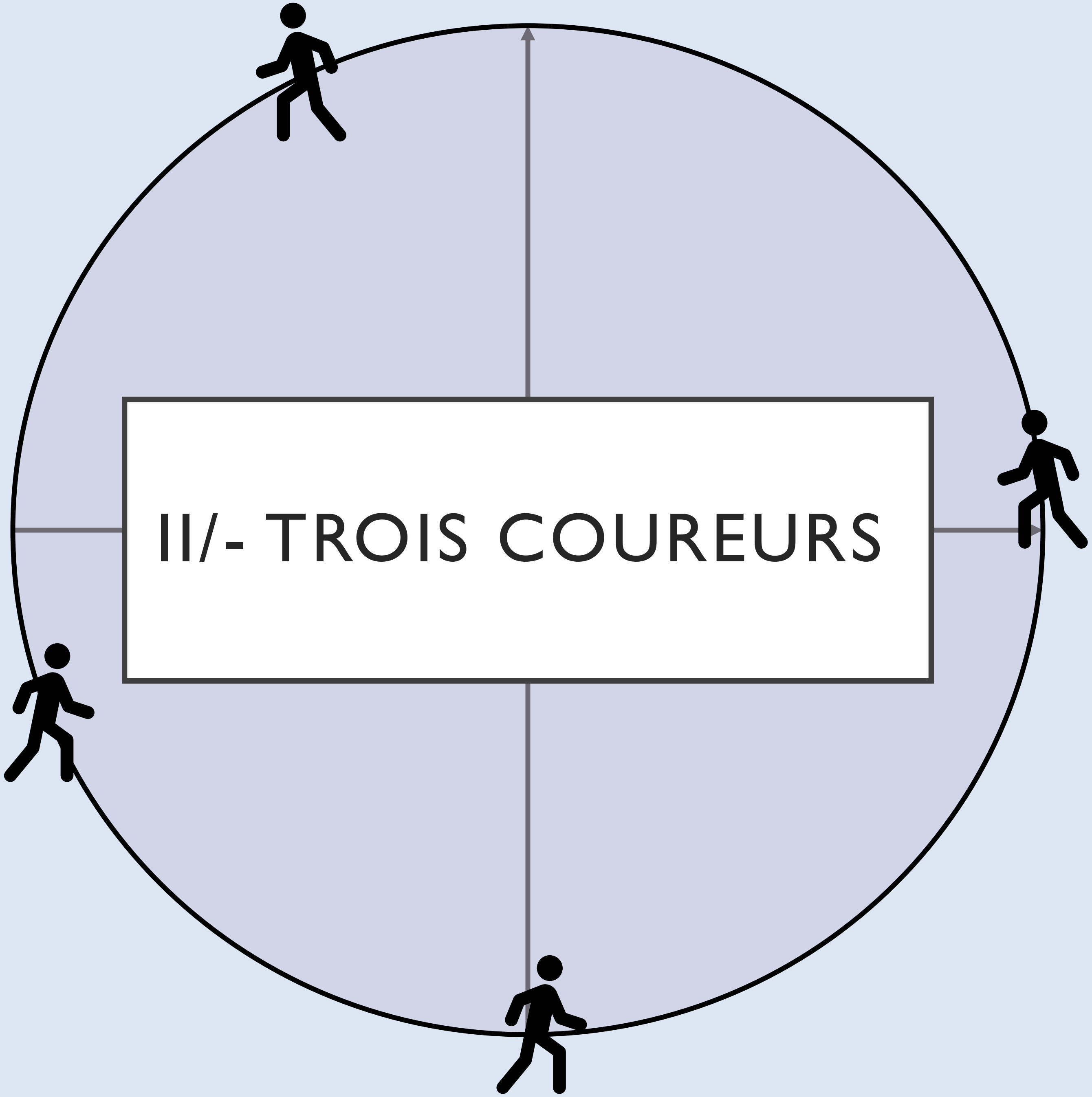
Si à l'instant n, D sépare $\boldsymbol{C_1}$ de $\boldsymbol{C_2}$, alors
à l'instant $n+1$, $\boldsymbol{C_1}$ aura parcouru :

$\boxed{\boldsymbol{d = p \times D}}$

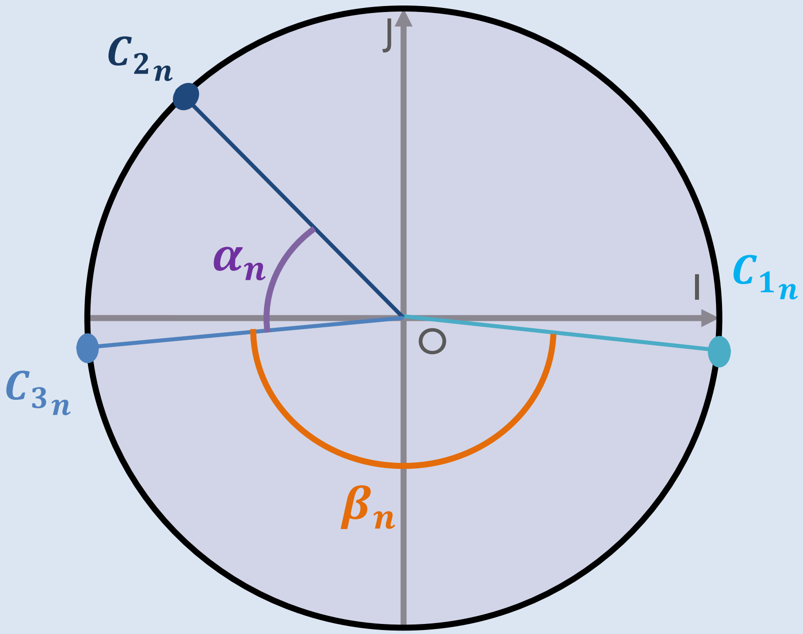
Avec $\boldsymbol{p} \in]\mathbf{0}; \mathbf{1}[$ fixé



$\boxed{\lim_{n\to\infty} \alpha_n = \boldsymbol{\pi}}$



II/- TROIS COUREURS



$$\boxed{\alpha_{n+1} = (1-p)\alpha_n + p\beta_n}$$

$$\boxed{\beta_{n+1} = -p\alpha_n + (1-2p)\beta_n + 2p\pi}$$

On conjecture que :

$$\boxed{\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = \frac{2\pi}{3}}$$

$$\boxed{\alpha_{n+1} = (1-p)\alpha_n + p\beta_n}$$

$$\boxed{\beta_{n+1} = -p\alpha_n + (1-2p)\beta_n + 2p\pi}$$

$$\boxed{\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \beta_n = \frac{2\pi}{3}}$$

$$\boxed{\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{2\pi}{3} \\ \frac{2\pi}{3} \end{pmatrix}}$$

$$\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix} = \begin{pmatrix} \frac{2\pi}{3} \\ \frac{2\pi}{3} \end{pmatrix}$$

$$\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

LA SUITE CONVERGE-T-ELLE VRAIMENT ?

$$\boxed{\alpha_{n+1} = (1-p)\alpha_n + p\beta_n}$$

$$\boxed{\beta_{n+1} = -p\alpha_n + (1-2p)\beta_n + 2p\pi}$$

$$\boxed{\lim_{n\to\infty}\begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

$$\boxed{M_{n+1} = \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix} = \begin{pmatrix} (1-p)\alpha_n + p\beta_n \\ -p\alpha_n + (1-2p)\beta_n \end{pmatrix} + \begin{pmatrix} 0 \\ 2p\pi \end{pmatrix}}$$

$$\boxed{\alpha_{n+1} = (1-p)\alpha_n + p\beta_n}$$

$$\boxed{\beta_{n+1} = -p\alpha_n + (1-2p)\beta_n + 2p\pi}$$

$$\boxed{\lim_{n\to\infty}\begin{pmatrix}\alpha_{n+1} - \frac{2\pi}{3}\\ \beta_{n+1} - \frac{2\pi}{3}\end{pmatrix} \overset{?}{=} \begin{pmatrix}0\\0\end{pmatrix}}$$

$$\boxed{M_{n+1} = \begin{pmatrix}\alpha_{n+1}\\ \beta_{n+1}\end{pmatrix} = \begin{pmatrix}(1-p)\alpha_n + p\beta_n\\ -p\alpha_n + (1-2p)\beta_n\end{pmatrix} + \begin{pmatrix}0\\ 2p\pi\end{pmatrix}}$$

$$\boxed{M_{n+1} = \begin{pmatrix}\alpha_{n+1}\\ \beta_{n+1}\end{pmatrix} = \begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}\begin{pmatrix}\alpha_n\\ \beta_n\end{pmatrix} + \begin{pmatrix}0\\ 2p\pi\end{pmatrix}}$$

$$\lim_{n\to\infty}\begin{pmatrix}\alpha_{n+1}-\frac{2\pi}{3}\\ \beta_{n+1}-\frac{2\pi}{3}\end{pmatrix}\overset{?}{=}\begin{pmatrix}0\\0\end{pmatrix}$$

$$M_{n+1}=\begin{pmatrix}\alpha_{n+1}\\ \beta_{n+1}\end{pmatrix}=\overbrace{\begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}}^{K}\overbrace{\begin{pmatrix}\alpha_n\\ \beta_n\end{pmatrix}}^{M_n}+\begin{pmatrix}0\\ 2p\pi\end{pmatrix}$$

$$\begin{pmatrix}\frac{2\pi}{3}\\ \frac{2\pi}{3}\end{pmatrix}=\begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}\begin{pmatrix}\frac{2\pi}{3}\\ \frac{2\pi}{3}\end{pmatrix}+\begin{pmatrix}0\\ 2p\pi\end{pmatrix}$$

$$M_{n+1} = \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix} = \begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix} \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix} + \begin{pmatrix} 0 \\ 2p\pi \end{pmatrix}$$

$$\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{2\pi}{3} \\ \frac{2\pi}{3} \end{pmatrix} = \begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix} \begin{pmatrix} \frac{2\pi}{3} \\ \frac{2\pi}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ 2p\pi \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} = \overbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}^{K} \overbrace{\begin{pmatrix} \alpha_n - \frac{2\pi}{3} \\ \beta_n - \frac{2\pi}{3} \end{pmatrix}}^{M_n{}'}$$

On note :

$$\widetilde{\alpha_n} = \alpha_n - \frac{2\pi}{3}$$
$$\widetilde{\beta_n} = \beta_n - \frac{2\pi}{3}$$

$$\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{n+1} - \frac{2\pi}{3} \\ \beta_{n+1} - \frac{2\pi}{3} \end{pmatrix} = \overbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}^{K} \overbrace{\begin{pmatrix} \alpha_n - \frac{2\pi}{3} \\ \beta_n - \frac{2\pi}{3} \end{pmatrix}}^{M_n{}'}$$

On note :

$$\boxed{\begin{aligned}\widetilde{\alpha_n} &= \alpha_n - \frac{2\pi}{3}\\ \widetilde{\beta_n} &= \beta_n - \frac{2\pi}{3}\end{aligned}}$$

$$\boxed{\lim_{n\to\infty} \begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} \stackrel{?}{=} \begin{pmatrix}0\\0\end{pmatrix}}$$

$$\boxed{\begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} = \overbrace{\begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}}^{K} \overbrace{\begin{pmatrix}\widetilde{\alpha_n}\\ \widetilde{\beta_n}\end{pmatrix}}^{M_n{}'}}$$

$$\boxed{\begin{pmatrix}\widetilde{\alpha_n}\\ \widetilde{\beta_n}\end{pmatrix} = K^n \begin{pmatrix}\widetilde{\alpha_0}\\ \widetilde{\beta_0}\end{pmatrix}}$$

$$\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}_{M_n'}$$

$$\lim_{n\to\infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

On cherche : $\lambda \epsilon \mathbb{C}$ *valeur propre de* K et $x, y \epsilon \mathbb{R}$ $(x,y) \neq (0,0)$ tel que :

$$K\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}_{M_n'}$$

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$$K\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix} = \begin{pmatrix} x - px + py \\ -px + y - 2py \end{pmatrix}$$

$$\boxed{\begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} = \overbrace{\begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}}^{K}\overbrace{\begin{pmatrix}\widetilde{\alpha_n}\\ \widetilde{\beta_n}\end{pmatrix}}^{M_n'}}$$

$$\boxed{\lim_{n\to\infty}\begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} \overset{?}{=} \begin{pmatrix}0\\0\end{pmatrix}}$$

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$$\boxed{\lambda : \quad 1-p+\frac{p}{s} = 1-2p-ps \iff ps^2+ps+p = 0}$$

$$\boxed{s = \frac{x}{y}}$$

$$\boxed{s_1 = \frac{-1-i\sqrt{3}}{2}} \qquad \boxed{s_2 = \frac{-1+i\sqrt{3}}{2}}$$

$$\boxed{\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \overbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}^{K} \overbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}^{M_n{}'}}$$

$$\boxed{\lim_{n\to\infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

On cherche : $\lambda \epsilon \mathbb{C}$ *valeur propre de* K et $x, y \epsilon \mathbb{R}$ $(x, y) \neq (0,0)$ tel que $K\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$

$$\boxed{\lambda_1 = \frac{2-3p-i\sqrt{3}}{2}} \qquad \boxed{\lambda_2 = \frac{2-3p+i\sqrt{3}}{2}}$$

$$\boxed{s_1 = \frac{-1-i\sqrt{3}}{2}} \qquad \boxed{s_2 = \frac{-1+i\sqrt{3}}{2}}$$

$$\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}_{M_n'}$$

$$\lim_{n\to\infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

On cherche : $\lambda \epsilon \mathbb{C}$ *valeur propre de* K et $x, y \epsilon \mathbb{R}$ $(x,y) \neq (0{,}0)$ tel que $K\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$

$$\lambda_1 = \frac{2-3p-i\sqrt{3}}{2} \qquad \lambda_2 = \frac{2-3p+i\sqrt{3}}{2}$$

Or $p \in]0;1[$ **Donc** $|\lambda| \epsilon [0,5;1[$

D'où $\lim_{n\to\infty} \lambda^n = 0$

p	0		0,5		1
$f'(\text{p})$		$-$	0	$+$	
$f(\text{p}) = \sqrt{1-3p+3p^2} = \lvert\lambda\rvert$	1	↘	0,5	↗	1

$$\boxed{\begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} = \underbrace{\begin{pmatrix}1-p & p\\ -p & 1-2p\end{pmatrix}}_{K}\underbrace{\begin{pmatrix}\widetilde{\alpha_n}\\ \widetilde{\beta_n}\end{pmatrix}}_{M_n'}}$$

$$\boxed{\lim_{n\to\infty}\begin{pmatrix}\widetilde{\alpha_{n+1}}\\ \widetilde{\beta_{n+1}}\end{pmatrix} \overset{?}{=} \begin{pmatrix}0\\ 0\end{pmatrix}}$$

$\lambda\epsilon\mathbb{C}$ *valeur propre de* K et $x, y\epsilon\mathbb{R}$ $(x, y) \neq (0,0)$ tel que $K\begin{pmatrix}x\\ y\end{pmatrix} = \lambda\begin{pmatrix}x\\ y\end{pmatrix}$

$$\boxed{\lambda_1 = \frac{2-3p-i\sqrt{3}}{2}} \qquad \boxed{\lambda_2 = \frac{2-3p+i\sqrt{3}}{2}}$$

$\lambda_1\epsilon\mathbb{C}$ *et* $\lambda_2\epsilon\mathbb{C}$ *sont valeurs propres de* K *si*

$\exists\, \vec{u} \in \mathbb{C}^2, \vec{u} \neq 0$ *tel que* $K\vec{u} = \lambda_1\vec{u}$

et $\vec{v} \in \mathbb{C}^2, \vec{v} \neq 0$ *tel que* $K\vec{v} = \lambda_2\vec{v}$

$$\boxed{\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}_{M_n'}}$$

$$\boxed{\lim_{n\to\infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

$\lambda \epsilon \mathbb{C}$ *est valeur propre de* K et $x, y \epsilon \mathbb{R}$ $(x, y) \neq (0,0)$ tel que $\boldsymbol{K} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \boldsymbol{\lambda} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}$

$$\boxed{\lambda_1 = \frac{2 - 3p - i\sqrt{3}}{2}}$$

$$\boxed{\lambda_2 = \frac{2 - 3p + i\sqrt{3}}{2}}$$

$\lambda_1 \epsilon \mathbb{C}$ *et* $\lambda_2 \epsilon \mathbb{C}$ *sont valeurs propres de* K *si*

$\exists\ \vec{u} \in \mathbb{C}^2, \vec{u} \neq 0$ *tel que* $K\vec{u} = \lambda_1 \vec{u}$

et $\vec{v} \in \mathbb{C}^2, \vec{v} \neq 0$ *tel que* $K\vec{v} = \lambda_2 \vec{v}$

$$\boxed{\begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1-p & p \\ -p & 1-2p \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix}}_{M_n'}}$$

$$\boxed{\lim_{n\to\infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \overset{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}}$$

$\lambda \epsilon \mathbb{C}$ *est valeur propre de* K et $x, y \epsilon \mathbb{R}$ $(x, y) \neq (0,0)$ tel que $K\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$

$$\boxed{\lambda \epsilon \mathbb{C} \text{ est valeur propre de } K \text{ si } \exists\, \vec{u} \in \mathbb{C}^2, \vec{u} \neq 0 \text{ tel que } K\vec{u} = \lambda\vec{u}}$$

$$\boxed{\lambda_1 : \qquad \vec{u} = \begin{pmatrix} \dfrac{2-3p-i\sqrt{3}}{2} \\ 1 \end{pmatrix}}$$

$$\boxed{\lambda_2 : \qquad \vec{v} = \begin{pmatrix} \dfrac{2-3p+i\sqrt{3}}{2} \\ 1 \end{pmatrix}}$$

$lemme : tout vecteur \ \vec{w}$
$\in \mathbb{C}^2 \ peut \ s'écrire :$

$$\vec{w} = a_1 \times \vec{u} + a_2 \times \vec{v}$$
$$(a_1, a_2 \epsilon \mathbb{C})$$

$$\lim_{n \to \infty} \begin{pmatrix} \widetilde{\alpha_{n+1}} \\ \widetilde{\beta_{n+1}} \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix} = \boldsymbol{K}^n \begin{pmatrix} \widetilde{\alpha_0} \\ \widetilde{\beta_0} \end{pmatrix}$$

$$\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix} = \boldsymbol{K}^n \vec{\boldsymbol{w}} = \boldsymbol{K}^n (a_1 \times \vec{u} + a_2 \times \vec{v})$$

$$\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix} = a_1 \times \boldsymbol{K}^n \times \vec{u} + a_2 \times \boldsymbol{K}^n \times \vec{v}$$

$$\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix} = a_1 \times \boldsymbol{\lambda_1}^n \times \vec{u} + a_2 \times \boldsymbol{\lambda_2}^n \times \vec{v}$$

$$\boldsymbol{\lambda_1}: \qquad \vec{\boldsymbol{u}} = \begin{pmatrix} \frac{2 - 3p - i\sqrt{3}}{2} \\ 1 \end{pmatrix}$$

$$\boldsymbol{\lambda_2}: \qquad \vec{\boldsymbol{v}} = \begin{pmatrix} \frac{2 - 3p + i\sqrt{3}}{2} \\ 1 \end{pmatrix}$$

$$\boldsymbol{K}^n \times \vec{u} = \boldsymbol{\lambda_1}^n \times \vec{u}$$

$$\boldsymbol{K}^n \times \vec{v} = \boldsymbol{\lambda_2}^n \times \vec{v}$$

$$\begin{pmatrix} \widetilde{\alpha_n} \\ \widetilde{\beta_n} \end{pmatrix} = a_1 \times \boldsymbol{\lambda_1}^{\boldsymbol{n}} \times \vec{u} + a_2 \times \boldsymbol{\lambda_2}^{\boldsymbol{n}} \times \vec{v}$$

$$\lim_{\boldsymbol{n}\to\infty} \boldsymbol{\lambda}^{\boldsymbol{n}} = \mathbf{0}$$

$$\lim_{\boldsymbol{n}\to\infty} \begin{pmatrix} \widetilde{\boldsymbol{\alpha}_{\boldsymbol{n}+\mathbf{1}}} \\ \widetilde{\boldsymbol{\beta}_{\boldsymbol{n}+\mathbf{1}}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

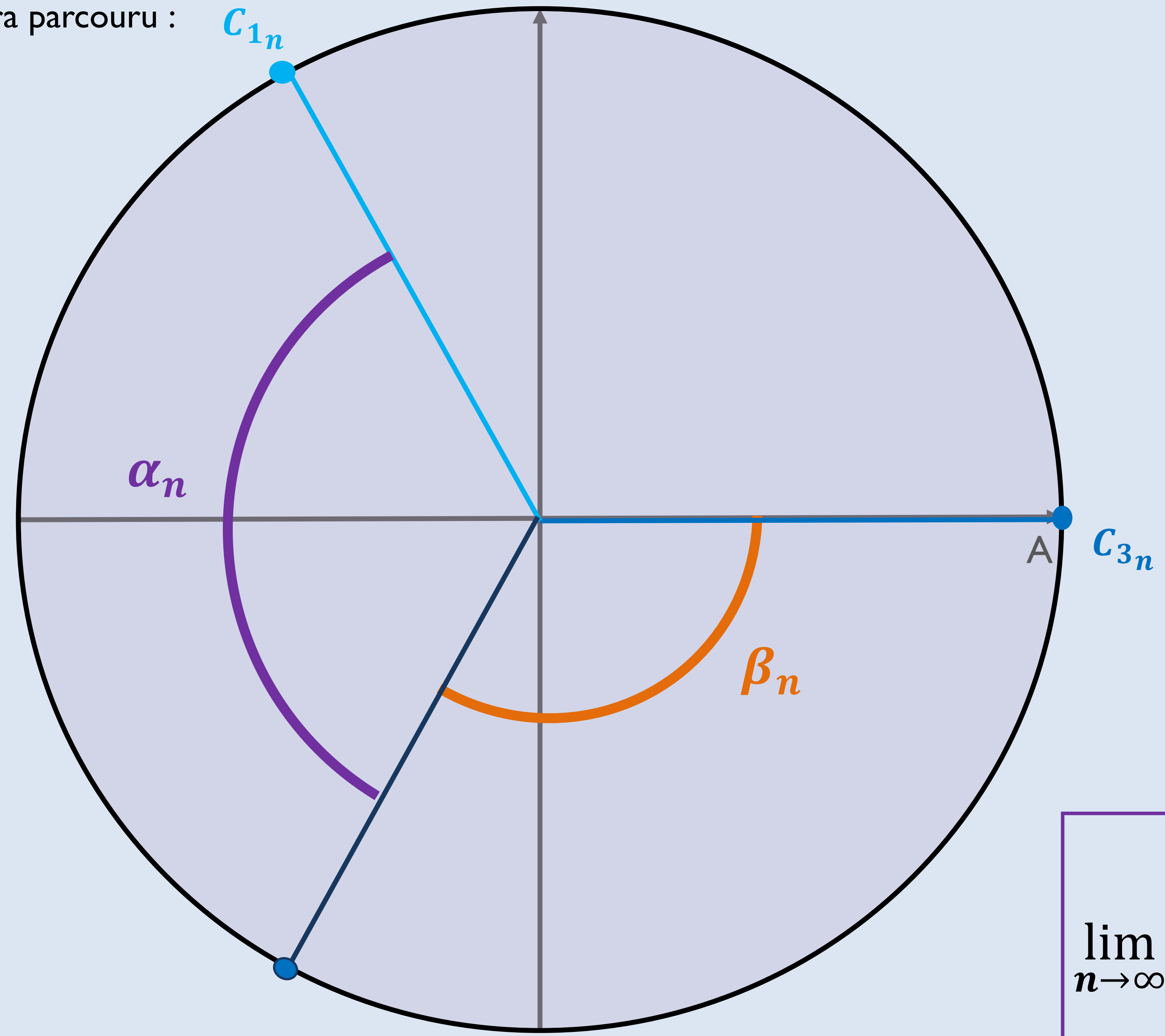
$$\lim_{\boldsymbol{n}\to\infty} \begin{pmatrix} \boldsymbol{\alpha}_{\boldsymbol{n}+\mathbf{1}} - \dfrac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}} \\ \boldsymbol{\beta}_{\boldsymbol{n}+\mathbf{1}} - \dfrac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

$$\lim_{\boldsymbol{n}\to\infty} \begin{pmatrix} \boldsymbol{\alpha}_{\boldsymbol{n}+\mathbf{1}} \\ \boldsymbol{\beta}_{\boldsymbol{n}+\mathbf{1}} \end{pmatrix} = \begin{pmatrix} \dfrac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}} \\ \dfrac{\mathbf{2}\boldsymbol{\pi}}{\mathbf{3}} \end{pmatrix}$$

Si à l'instant n, D sépare $\boldsymbol{C_1}$ de $\boldsymbol{C_2}$, alors
à l'instant $n+1$, $\boldsymbol{C_1}$ aura parcouru :

$$\boxed{\boldsymbol{d = p \times D}}$$

Avec $\boldsymbol{p} \in \left]\mathbf{0}; \mathbf{1}\right[$ fixé



$$\lim_{n\to\infty} \begin{pmatrix} \alpha_{n+1} \\ \beta_{n+1} \end{pmatrix} = \begin{pmatrix} \dfrac{2\pi}{3} \\ \dfrac{2\pi}{3} \end{pmatrix}$$

III/- CONJECTURE

$$\boxed{\lim_{n\to\infty} \boldsymbol{angles} = \frac{2\pi}{\boldsymbol{nbre\ de\ coureurs}}}$$

Avec $\boldsymbol{angles\ initiaux} \neq 0$ (et pour $\boldsymbol{p \neq 0\ et\ p \neq 1}$)

MERCI DE NOUS AVOIR ÉCOUTÉES

Avez-vous des questions ?