Correction des exercices sur les identités remarquables

I

Développer en utilisant les identités remarquables :

$A = (x-5)^2 x^2 - 2 \times x \times 5 + 5^2 = \boxed{x^2 - 10x + 25}$

$B = (4-2x)^2 = 4^2 - 2 \times 4 \times 2x + (2x)^2 = \boxed{16 - 16x + 4x^2 = 4x^2 - 16x + 16}$

$C = \left(\frac{1}{2}x + 1\right)^2 = \left(\frac{1}{2}x\right)^2 + 2 \times \frac{1}{2}x \times 1 + 1^1 = \boxed{\frac{1}{4}x^2 + x + 1}$

$D = (2x-7)(2x+7) = (2x)^2 - 7^2 = \boxed{4x^2 - 49}$

$E = \left(\frac{1}{3}x - 4\right)\left(\frac{1}{3}x + 4\right) = \left(\frac{1}{3}x\right)^2 - 4^2 = \boxed{\frac{1}{9}x^2 - 16}$

$F = \left(2x - \sqrt{3}\right)\left(2x + \sqrt{3}\right) = (2x^2) - \sqrt{3}^2 = \boxed{4x^2 - 3}$

$G = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = \boxed{x^2 + 2 + \frac{1}{x^2}}$

$H = \left(\sqrt{3} + \sqrt{2}\right)\left(\sqrt{3} - \sqrt{2}\right) = \sqrt{3}^2 - \sqrt{2}^2 = 3 - 2 = \boxed{1}$

$I = (7-2x)(2x-7) = -(2x-7)(2x+7) = -\left[(2x)^2 - 7^2\right] = -\left(4x^2 - 49\right) = \boxed{49 - 4x^2}$

II

Développer et simplifier les expressions suivantes :

$A = \left(\sqrt{7} - 3\right)\left(\sqrt{7} + 3\right) = \sqrt{7}^2 - 3^2 = 7 - 9 = \boxed{-2}$

$B = \left(\sqrt{3} + \sqrt{5}\right)^2 + \left(\sqrt{15} - 1\right)^2$
$= \left[\sqrt{3}^2 + 2 \times \sqrt{3} \times \sqrt{5} + \sqrt{5}^2\right] + \left[\sqrt{15}^2 - 2 \times \sqrt{15} \times 1 + 1^2\right]$
$= \left[3 + 2\sqrt{15} + 5\right] + \left[15 - 2\sqrt{15} + 1\right]$
$= 8 + 2\sqrt{15} + 16 - 2\sqrt{15} = \boxed{24}$

$C = \left(\sqrt{4 - \sqrt{7}} + \sqrt{4 + \sqrt{7}}\right)^2$
$= \sqrt{4 - \sqrt{7}}^2 + 2 \times \sqrt{4 - \sqrt{7}} \times \sqrt{4 + \sqrt{7}} + \sqrt{4 + \sqrt{7}}^2$
$= 4 - \sqrt{7} + 2\sqrt{\left(4 - \sqrt{7}\right)\left(4 + \sqrt{7}\right)} + 4 + \sqrt{7}$
$= 4 - \sqrt{7} + 4 + \sqrt{7} + 2\sqrt{4^2 - \sqrt{7}^2}$
$= 8 + 2\sqrt{16 - 7} = 8 + 2\sqrt{9} = 8 + 2 \times 3 = \boxed{14}$