

2^{nde} : correction du TD n° 13 sur les développements et identités remarquables

Exercice I

$$a(b-c) + b(c-a) + c(a-b) = ab - ac + bc - ab + ac - bc = \boxed{0}$$

Exercice II

Développer les expressions suivantes :

$$A(x) = (-4x-2)(6x-1) = -24x^2 + 4x - 12x + 2 = \boxed{-24x^2 - 8x + 2}$$

$$B(x) = (-4x-5)(3x+5) = -12x^2 - 20x - 15x - 25 = \boxed{-12x^2 - 35x - 25}$$

$$C(x) = (4-5x)(2x+7) = 8x + 28 - 10x^2 - 35x = -10x^2 = \boxed{-10x^2 - 27x + 28}$$

$$D(x) = 6(7-9x)(-8x-9) = 6[-56x - 63 + 72x^2 + 81x] = 6(72x^2 + 25x - 63) = \boxed{432x^2 + 150x - 378}$$

$$E(x) = -5(-9x-5)(-6x-10) = -5[54x^2 + 90x + 30x + 50] = -5(54x^2 + 120x + 50) = \boxed{-270x^2 - 600x - 250}$$

$$\begin{aligned} F(x) &= (5x+5)(8x-5) + (-10x-2)(9-6x) = [40x^2 - 25x + 40x - 25] + [-90x + 60x^2 - 18 + 12x] \\ &= [40x^2 - 15x - 25] + [60x^2 - 78x - 18] \\ &= 40x^2 + 15x - 25 + 60x^2 - 78x - 18 \\ &= \boxed{100x^2 - 63x - 43} \end{aligned}$$

Exercice III

$$1. (\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - \sqrt{2}^2 = 3 - 2 = \boxed{1}$$

$$2. (a+b)^3 = (a+b)(a+b)^2 = (a+b)(a^2 + 2ab + b^2) = a^3 + 2ab^2 + ab^2 + a^2b + 2ab^2 + b^3 = \boxed{a^3 + 3a^2b + 3ab^2 + b^3}$$

$$3. (a-b)^3 = (a+(-b))^3 = a^3 + 3a^2 \times (-b) + 3a \times (-b)^2 + (-b)^3 = \boxed{a^3 - 3a^2b + 3ab^2 - b^3}$$

$$4. (a-b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 = \boxed{a^3 - b^3}$$

Exercice IV

Développer les expressions suivantes :

$$A = (\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = \sqrt{x^2} - \sqrt{y^2} = \boxed{x - y}$$

$$B = (x^2 + y^2)^2 = (x^2)^2 + 2x^2y^2 + (y^2)^2 = \boxed{x^4 + 2x^2y^2 + y^4}$$

$$\begin{aligned} C &= \left(x + \frac{1}{x}\right)^2 \text{ (pour tout } x \neq 0) \\ &= x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2} \end{aligned}$$

Exercice V

Développer et réduire les expressions suivantes :

$$A(x) = (x+4)^2 - (x+3)(x-4)$$

$$B(x) = (x+4)(2x+5) + (2x-1)^2$$

$$C(x) = (4x-7)^2 - (3x+1)^2$$