Correction de la feuille d'exercices sur les racines carrées

I

Écrire plus simplement :

$A = \sqrt{3} \times 4\sqrt{3} = 4 \times \left(\sqrt{3}\right)^2 = 4 \times 3 = 12$; $\boxed{A = 12}$

$B = \left(2\sqrt{7}\right) \times \left(-5\sqrt{7}\right) = 2 \times (-5) \times \left(\sqrt{7}\right)^2 = -10 \times 7 = -70$; $\boxed{B = -70}$

$C = \left(3\sqrt{7}\right)^2 = 3^3 \times \sqrt{7}^2 := 9 \times 7 = 63$; $\boxed{C = 63}$

$D = \left(-2\sqrt{5}\right) \times \left(-4\sqrt{5}\right) = (-2) \times (-4) \times \left(\sqrt{5}\right)^2 = 8 \times 5 = 40$; $\boxed{D = 40}$

II

Calculer de manière la plus simple possible :

$A = \sqrt{4 \times 9 \times 16} = \sqrt{4} \times \sqrt{9} \times \sqrt{16} = 2 \times 3 \times 4 = 24$; $\boxed{A = 24}$

$B = \sqrt{2^2 \times 3^4 \times 5^2} = \sqrt{2^2} \times \sqrt{3^4} \times \sqrt{5^2} = 2 \times 3^2 \times 5 = 80$; $\boxed{B = 80}$

$C = \sqrt{144 \times 225} = \sqrt{144} \times \sqrt{225} = 12 \times 15 = 180$; $\boxed{C = 180}$

$D = \sqrt{\dfrac{225 \times 121}{36 \times 144}} = \dfrac{\sqrt{225 \times 121}}{\sqrt{36} \times \sqrt{144}} = \dfrac{\sqrt{225} \times \sqrt{121}}{\sqrt{36} \times \sqrt{144}} = \dfrac{15 \times 11}{6 \times 12} = \dfrac{\cancel{3} \times 5 \times 11}{\cancel{3} \times 2 \times 12} = \dfrac{55}{24}$; $\boxed{D = \dfrac{55}{24}}$

III

Écrire les nombres suivants sous la forme $a\sqrt{b}$, a et b étant entiers, b le plus petit possible.

$A = \sqrt{50} = \sqrt{25 \times 2} = \sqrt{5^2 \times 2} = 5\sqrt{2}$; $\boxed{A = 5\sqrt{2}}$

$B = \sqrt{245} = \sqrt{49 \times 5} = \sqrt{7^2 \times 5} = 7\sqrt{5}$; $\boxed{B = 7\sqrt{5}}$

$C = \sqrt{20} \times \sqrt{15} = \sqrt{2^2 \times 5} \times \sqrt{5} \times \sqrt{3} = 2\sqrt{5} \times \sqrt{5} \times \sqrt{3} = 2 \times \left(\sqrt{5}\right)^2 \times \sqrt{3} = 2 \times 5 \times \sqrt{3} = 10\sqrt{3}$; $\boxed{C = 10\sqrt{3}}$

$D = \sqrt{108} = \sqrt{36 \times 3} = \sqrt{6^2 \times 3} = 6\sqrt{3}$; $\boxed{D = 6\sqrt{3}}$

$E = 5\sqrt{18} = 5 \times \sqrt{9 \times 2} = 5 \times \sqrt{3^2 \times 2} = 5 \times 3\sqrt{2} = 15\sqrt{2}$; $\boxed{E = 15\sqrt{2}}$

$F = \sqrt{32} \times \sqrt{14} = \sqrt{16 \times 2} \times \sqrt{2} \times \sqrt{7} = 4\sqrt{2} \times \sqrt{2} \times \sqrt{7} = 4 \times 2 \times \sqrt{7} = 8\sqrt{7}$; $\boxed{F = 8\sqrt{7}}$

$G = \sqrt{1\,000} = \sqrt{10^2 \times 10} = 10\sqrt{10}$; $\boxed{G = 10\sqrt{10}}$

$H = 2\sqrt{147} = 2 \times \sqrt{7^2 \times 3} = 2 \times 7\sqrt{3} = 14\sqrt{3}$; $\boxed{H = 14\sqrt{3}}$

IV

Écrire plus simplement :

$A = \sqrt{98}+\sqrt{32}-\sqrt{8} = \sqrt{49\times 2}+\sqrt{16\times 2}-\sqrt{4\times 2} = \sqrt{7^2\times 2}+\sqrt{4^2\times 2}-\sqrt{2^2\times 2} = 7\sqrt{2}+4\sqrt{2}-2\sqrt{2} = (7+4-2)\sqrt{2}$
$= \boxed{9\sqrt{2}}$.

$B = \sqrt{24}-\sqrt{96}+3\sqrt{54} = \sqrt{4\times 6}-\sqrt{16\times 6}+3\sqrt{9\times 6} = \sqrt{2^2\times 6}-\sqrt{4^2\times 6}+3\sqrt{3^2\times 6} = 2\sqrt{6}-4\sqrt{6}+3\times 3\sqrt{6} =$
$(2-4+9)\sqrt{6} = \boxed{7\sqrt{6}}$

$C = \sqrt{20}-\sqrt{5}-\sqrt{45} = \sqrt{4\times 5}-\sqrt{5}-\sqrt{9\times 5} = 2\sqrt{5}-\sqrt{5}-3\sqrt{5} = \boxed{-2\sqrt{5}}$

V

Comparer, sans utiliser la calculatrice, les nombres suivants :

a) $6\sqrt{2}$ et $5\sqrt{3}$
$6\sqrt{2} = \sqrt{6^2\times 2} = \sqrt{72}$ et $5\sqrt{3} = \sqrt{5^2\times 3} = \sqrt{75}$.
$72 < 75$ donc $\sqrt{72} < \sqrt{75}$ donc $\boxed{6\sqrt{2} < 5\sqrt{3}}$

b) $7\sqrt{2}$ et $3\sqrt{11}$.
$7\sqrt{2} = \sqrt{49\times 2} = \sqrt{98}$; $3\sqrt{11} = \sqrt{3^2\times 11} = \sqrt{99} > \sqrt{98}$ donc $\boxed{7\sqrt{2} < "\sqrt{11}}$

c) $-3\sqrt{7}$ et $5\sqrt{2}$: $-3\sqrt{7} < 0$ et $5\sqrt{2} > 0$ donc $\boxed{-3\sqrt{7} < 5\sqrt{2}}$ (sans calcul !)

VI Brevet Aix-Marseille juin 2005

a) $C = 4\sqrt{7}-8\sqrt{28}+\sqrt{700} = 4\sqrt{7}-8\times 2\sqrt{7}+10\sqrt{7} = (4-8+10)\sqrt{7} = \boxed{6\sqrt{7}}$.

b) $\left(4\sqrt{5}+2\right)^2 = \left(4\sqrt{5}+2\right)\left(4\sqrt{5}+2\right) = 4\sqrt{5}\times 4\sqrt{5}+4\sqrt{5}\times 2+2\times 4\sqrt{5}+2^2 = 16\times 5+8\sqrt{5}+8\sqrt{5}+4 = \boxed{84+16\sqrt{5}}$
(en appliquant la double distributivité).
ou avec une identité remarquable :
$\left(4\sqrt{5}+2\right)^2 = \left(4\sqrt{5}\right)^2+2\times 4\sqrt{5}\times 2+2^2 = 16\times 5+8\sqrt{5}+4 = \boxed{84+8\sqrt{5}}$

VII Brevet Antilles–Guyane juin 2005

$C = \left(3-4\sqrt{5}\right)\left(3+4\sqrt{5}\right) = 3^2-\left(4\sqrt{5}\right)^2 = 9-16\times 5 = 9-80 = \boxed{-71\in\mathbb{Z}}$ (en appliquant une identité remarquable : $(a+b)(a-b) = a^2-b^2$)