TS Correction des exercices n° 11, 51 b, 52 a et 53 pp 280 − 282

(11) a) $2iz = 3+i \iff z = \dfrac{3+i}{2i} \iff z = \dfrac{i(3+i)}{2i^2} = \dfrac{3i-1}{-2} = \dfrac{1}{2} - \dfrac{3}{2}i$

b) $2z - 3 = iz + 4 \iff 2z - iz = 3+4 \iff (2-i)z = 7 \iff z = \dfrac{7}{2-i}$,

d'où $2z-3 = iz+4 \iff z = \dfrac{7(2+i)}{(2-i)(2+i)} = \dfrac{14+7i}{2^2-i^2} = \dfrac{14+7i}{4+1} = \dfrac{14}{5} + \dfrac{7}{5}i$.

(51) b) $(4+i)\bar{z} = 1-i \iff \bar{z} = \dfrac{1-i}{4+i} = \dfrac{(1-i)(4-i)}{(4+i)(4-i)} = \dfrac{4-i-4i-1}{4^2-i^2} = \dfrac{3-5i}{16+1}$,

d'où $(4+i)\bar{z} = 1-i \iff \bar{z} = \dfrac{3}{17} - \dfrac{5}{17}i \iff z = \dfrac{3}{17} + \dfrac{5}{17}i$.

(52) a) $(2+4i)z + 3 - i = 5z - i \iff 3 = 5z - (2+4i)z \iff (3-4i)z = 3$, d'où

$(2+4i)z + 3 - i = 5z - i \iff z = \dfrac{3}{3-4i} = \dfrac{3(3+4i)}{(3-4i)(3+4i)} = \dfrac{9+12i}{9+16} = \dfrac{9}{25} + \dfrac{12}{25}i$

(53) a) Notons (E) : $(2-i)z + 3 - i = 3\bar{z} - i$ et posons $z = a+bi$ avec $a \in \mathbb{R}$ et $b \in \mathbb{R}$.

On a alors $(E) \iff (2-i)(a+bi) + 3 - i = 3(a-bi) - i$, d'où

$(E) \iff 2a + 2bi - ai + b + 3 = 3a - 3bi$

$(E) \iff \underbrace{2a+b+3}_{\in\mathbb{R}} + \underbrace{(2b-a)}_{\in\mathbb{R}}i = \underbrace{3a}_{\in\mathbb{R}} \underbrace{-3b}_{\in\mathbb{R}}i$

$(E) \iff \begin{cases} 2a+b+3 = 3a \\ \text{et} \\ 2b - a = -3b \end{cases}$ (par unicité de la forme algébrique d'un nombre complexe)

$(E) \iff \begin{cases} a = b+3 \\ \text{et} \\ 5b - a = 0 \end{cases} \iff \begin{cases} a = b+3 \\ \text{et} \\ 5b - (b+3) = 0 \end{cases} \iff \begin{cases} a = b+3 \\ \text{et} \\ 4b = 3 \end{cases} \iff \begin{cases} a = \frac{3}{4} + 3 = \frac{15}{4} \\ \text{et} \\ b = \frac{3}{4} \end{cases} \iff z = \dfrac{15}{4} + \dfrac{3}{4}i$

b) Notons (E) : $4 - iz + 3 - i = 2(\bar{z} - 2i)$ et posons $z = a+bi$ avec $a \in \mathbb{R}$ et $b \in \mathbb{R}$.

On a alors $(E) \iff 7 - i - iz = 2\bar{z} - 4i \iff 7 - i - i(a+bi) = 2(a-bi) - 4i$

$(E) \iff 7 - i - ai + b = 2a - 2bi - 4i \iff \underbrace{7+b}_{\in\mathbb{R}} + \underbrace{(-1-a)}_{\in\mathbb{R}}i = \underbrace{2a}_{\in\mathbb{R}} + \underbrace{(-2b-4)}_{\in\mathbb{R}}i$,

d'où par unicité de la forme algébrique d'un nombre complexe :

$(E) \iff \begin{cases} 7+b = 2a \\ \text{et} \\ -1-a = -2b-4 \end{cases} \iff \begin{cases} b = 2a-7 \\ \text{et} \\ 2b - a = -3 \end{cases} \iff \begin{cases} b = 2a-7 \\ \text{et} \\ 2(2a-7) - a = -3 \end{cases} \iff \begin{cases} b = 2a-7 \\ \text{et} \\ 4a - 14 - a = -3 \end{cases}$

$\iff \begin{cases} b = 2a-7 \\ \text{et} \\ 3a = 11 \end{cases} \iff \begin{cases} b = 2 \times \frac{11}{3} - 7 = \frac{22}{3} - \frac{21}{3} = \frac{1}{3} \\ \text{et} \\ a = \frac{11}{3} \end{cases} \iff z = \dfrac{11}{3} + \dfrac{1}{3}i$