1S1 – Correction du contrôle n° : 3 de MATHÉMATIQUES 2014/2015

$\boxed{0}$ 1) • $20° = 20 \times \frac{\pi}{180}$ rad $= \frac{\pi}{9}$ rad donc les réels auxquels sont associés A sont les $\frac{\pi}{9} + 2k\pi$, $k \in \mathbb{Z}$

• $78° = 78 \times \frac{\pi}{180}$ rad $= \frac{13\pi}{30}$ rad et $\frac{\pi}{2} + \frac{13\pi}{30} = \frac{14\pi}{15}$

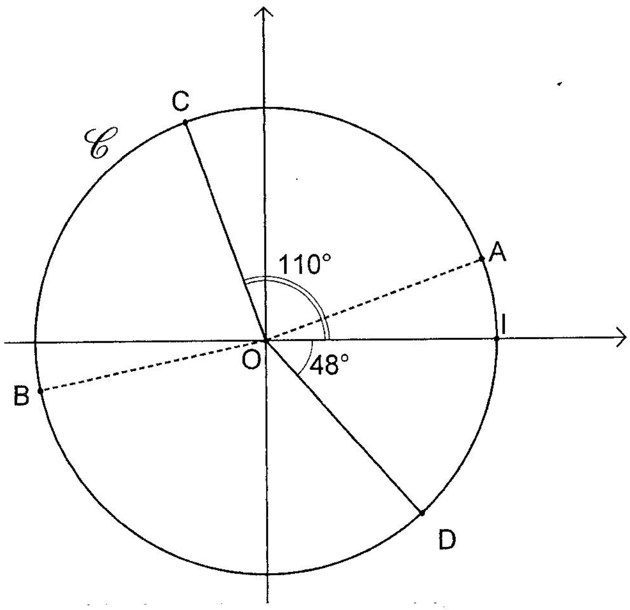
donc les réels auxquels sont associés B sont les :

$$-\frac{14\pi}{15} + 2k\pi \ , k \in \mathbb{Z}$$

2) • $\frac{11\pi}{18}$ rad $= \frac{180}{\pi} \times \frac{11\pi}{18}$ deg $= 110°$

• $\frac{4\pi}{15}$ rad $= \frac{180}{\pi} \times \frac{4\pi}{15}$ deg $= 48°$

d'où →



$\boxed{1}$ • $\frac{43\pi}{3} = \frac{\pi + 42\pi}{3} = \frac{\pi}{3} + 14\pi = \frac{\pi}{3} + 7 \times 2\pi$

donc $\sin \frac{43\pi}{3} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

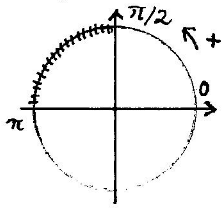
• $\frac{77\pi}{6} = \frac{5\pi + 72\pi}{6} = \frac{5\pi}{6} + 12\pi = \frac{5\pi}{6} + 6 \times 2\pi$, d'où $\cos \frac{77\pi}{6} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

• $-\frac{27\pi}{4} = \frac{-3\pi - 24\pi}{4} = -\frac{3\pi}{4} - 6\pi = -\frac{3\pi}{4} - 3 \times 2\pi$, d'où $\sin\left(-\frac{27\pi}{4}\right) = \sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\boxed{2}$ On a $\sin^2\theta + \cos^2\theta = 1$ donc $\cos^2\theta = 1 - \left(\frac{5}{13}\right)^2 = \frac{169 - 25}{169} = \frac{144}{169}$,

d'où $\cos\theta = \sqrt{\frac{144}{169}} = \frac{12}{13}$ ou $\cos\theta = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$

et finalement $\cos\theta = -\frac{12}{13}$ car $\cos\theta \leq 0$, vu que $\theta \in [\frac{\pi}{2} ; \pi]$



$\boxed{3}$ a) Avant toute chose, remarquons que $36° = \frac{\pi}{180} \times 36$ rad $= \frac{\pi}{5}$ rad. On a donc $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$.

D'autre part, $\cos\frac{4\pi}{5} = \cos\left(\pi - \frac{\pi}{5}\right) = -\cos\frac{\pi}{5} = -\frac{\sqrt{5}+1}{4}$,

$\cos\frac{6\pi}{5} = \cos\left(\pi + \frac{\pi}{5}\right) = -\cos\frac{\pi}{5} = -\frac{\sqrt{5}+1}{4}$,

$\sin\frac{3\pi}{10} = \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$,

$\sin\frac{7\pi}{10} = \sin\left(\pi - \frac{3\pi}{10}\right) = \sin\frac{3\pi}{10} = \frac{\sqrt{5}+1}{4}$ et

$\sin\frac{13\pi}{10} = \sin\left(\pi + \frac{3\pi}{10}\right) = -\sin\frac{3\pi}{10} = -\frac{\sqrt{5}+1}{4}$.

RAPPELS :

$\cos(\pi - \alpha) = -\cos\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\cos(\pi + \alpha) = -\cos\alpha$

$\sin(\pi + \alpha) = -\sin\alpha$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

b) On a $\frac{2014\pi}{5} = \frac{4\pi + 2010\pi}{5} = \frac{4\pi}{5} + 402\pi = \frac{4\pi}{5} + 201 \times 2\pi$ donc $\cos\left(\frac{2014\pi}{5}\right) = \cos\frac{4\pi}{5} = -\frac{\sqrt{5}+1}{4}$,

et $\frac{2013\pi}{10} = \frac{-7\pi + 2020\pi}{10} = -\frac{7\pi}{10} + 101 \times 2\pi$ donc $\sin\left(\frac{2013\pi}{10}\right) = \sin\left(-\frac{7\pi}{10}\right) = -\sin\frac{7\pi}{10} = -\frac{\sqrt{5}+1}{4}$

(car $\sin(-\alpha) = -\sin\alpha$)

$\boxed{4}$ $8\sin^2 x = 5 - 6\sin x \iff 8\sin^2 x + 6\sin x - 5 = 0$

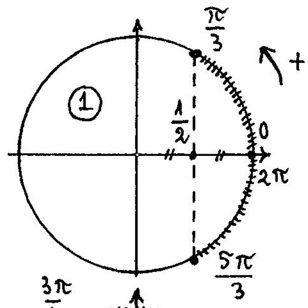
$\iff 8X^2 + 6X - 5 = 0$, en posant $X = \sin x$. Cette dernière équation est du second degré, a pour discriminant $\Delta = 196 > 0$ et possède donc deux solutions :

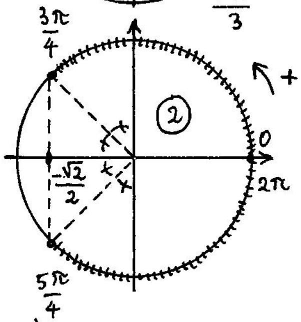
$X_1 = \frac{-6 - \sqrt{196}}{2 \times 8} = \frac{-20}{16} = -\frac{5}{4}$ et $X_2 = \frac{-6 + \sqrt{196}}{2 \times 8} = \frac{8}{16} = \frac{1}{2}$

Notre équation initiale équivaut donc à : $\left(\underbrace{\sin x = -\frac{5}{4}}_{\text{impossible car } -\frac{5}{4} \notin [-1;1]} \quad \text{ou} \quad \sin x = \underbrace{\frac{1}{2}}_{\sin\frac{\pi}{6}}\right)$

$\iff \sin x = \sin\frac{\pi}{6}$

$\iff \left(x = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \quad \text{ou} \quad x = \underbrace{\pi - \frac{\pi}{6}}_{\frac{5\pi}{6}} + 2k\pi,\ k \in \mathbb{Z}\right)$





$\boxed{5}$ a) $\cos x \geq \frac{1}{2} \iff x \in \left[0;\frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3};2\pi\right[$ (voir cercle ①)

b) $\cos x \geq -\frac{\sqrt{2}}{2} \iff x \in \left[0;\frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4};2\pi\right[$ (voir cercle ②)

c) Le discriminant vaut $\Delta = [2(\sqrt{2}-1)]^2 - 4 \times 4 \times (-\sqrt{2})$, soit

$\Delta = 4(\sqrt{2}^2 - 2\sqrt{2} + 1^2) + 16\sqrt{2} = 4(3-2\sqrt{2}) + 16\sqrt{2} = 12 + 8\sqrt{2} > 0.$

Or $[2(1+\sqrt{2})]^2 = 4(1^2 + 2\sqrt{2} + \sqrt{2}^2) = 4(3+2\sqrt{2}) = 12 + 8\sqrt{2}$,

ce qui prouve bien que $\Delta = [2(1+\sqrt{2})]^2$. On en déduit

que $\sqrt{\Delta} = 2(1+\sqrt{2})$, car $2(1+\sqrt{2}) > 0$.

Les racines du trinôme T sont alors :

$X_1 = \frac{-2(\sqrt{2}-1) - 2(1+\sqrt{2})}{2 \times 4} = \frac{-4\sqrt{2}}{8} = -\frac{\sqrt{2}}{2}$ et $X_2 = \frac{-2(\sqrt{2}-1) + 2(1+\sqrt{2})}{2 \times 4} = \frac{4}{8} = \frac{1}{2}.$

On en déduit que $T = 4(X - X_1)(X - X_2) = 4\left(X + \frac{\sqrt{2}}{2}\right)\left(X - \frac{1}{2}\right)$

d) $4\cos^2 x + 2(\sqrt{2}-1)\cos x - \sqrt{2} \leq 0 \iff 4\left(\cos x + \frac{\sqrt{2}}{2}\right)\left(\cos x - \frac{1}{2}\right) \leq 0$

$\iff \left(\cos x + \frac{\sqrt{2}}{2}\right)\left(\cos x - \frac{1}{2}\right) \leq 0$ car $4 > 0$

Or $\cos x + \frac{\sqrt{2}}{2} \geq 0 \iff \cos x \geq -\frac{\sqrt{2}}{2} \iff x \in \left[0;\frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4};2\pi\right[$ (voir b))

et $\cos x - \frac{1}{2} \geq 0 \iff \cos x \geq \frac{1}{2} \iff x \in \left[0;\frac{\pi}{3}\right] \cup \left[\frac{5\pi}{3};2\pi\right[$ (voir a)), d'où :

x	0		$\frac{\pi}{3}$		$\frac{3\pi}{4}$		$\frac{5\pi}{4}$		$\frac{5\pi}{3}$		2π
$\cos x + \frac{\sqrt{2}}{2}$		$+$		$+$	0	$-$	0	$+$		$+$	
$\cos x - \frac{1}{2}$		$+$	0	$-$		$-$		$-$	0	$+$	
$\left(\cos x + \frac{\sqrt{2}}{2}\right)\left(\cos x - \frac{1}{2}\right)$		$+$	0	$-$	0	$+$	0	$-$	0	$+$	

Ainsi, $4\cos^2 x + 2(\sqrt{2}-1)\cos x - \sqrt{2} \leq 0 \iff x \in \left[\frac{\pi}{3};\frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4};\frac{5\pi}{3}\right]$

$\boxed{6}$ a) $(\overrightarrow{AB},\overrightarrow{AC}) = \frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$ b) $(\overrightarrow{BC},\overrightarrow{BA}) = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}$

c) $(\overrightarrow{DC},\overrightarrow{DA}) = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$ d) $(\overrightarrow{AF},\overrightarrow{EB}) = \pi + 2k\pi,\ k \in \mathbb{Z}$

e) $(\overrightarrow{AD},\overrightarrow{AC}) = -\frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$ f) $(\overrightarrow{FE},\overrightarrow{FB}) = \frac{\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$

g) $(\overrightarrow{BC},\overrightarrow{AF}) = (\overrightarrow{BC},\overrightarrow{BA}) + (\overrightarrow{BA},\overrightarrow{AF}) + 2k\pi,\ k \in \mathbb{Z}$ car $\overrightarrow{AF} = \overrightarrow{BE}$

$= \frac{\pi}{6} + (\overrightarrow{BA},\overrightarrow{BE}) + 2k\pi,\ k \in \mathbb{Z}$

$= \frac{\pi}{6} + \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$

$= \frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z}$

$\boxed{6}$ h) $$(\overrightarrow{EB},\overrightarrow{AD}) = \underbrace{(\overrightarrow{EB},\overrightarrow{BE})}_{\pi} + (\overrightarrow{BE},\overrightarrow{BA}) + \underbrace{(\overrightarrow{BA},\overrightarrow{AB})}_{\pi} + (\overrightarrow{AB},\overrightarrow{AC}) + (\overrightarrow{AC},\overrightarrow{AD}) + 2k\pi,\ k\in\mathbb{Z}$$

$$= 2\pi - \frac{\pi}{2} + \frac{\pi}{3} + \frac{\pi}{4} + 2k\pi,\ k\in\mathbb{Z}$$

$$= \frac{\pi}{12} + 2k\pi,\ k\in\mathbb{Z}$$

car $\overrightarrow{AB} = \overrightarrow{FE}$

i) $$(\overrightarrow{DC},\overrightarrow{FB}) = (\overrightarrow{DC},\overrightarrow{DA}) + \underbrace{(\overrightarrow{DA},\overrightarrow{AD})}_{\pi} + (\overrightarrow{AD},\overrightarrow{AC}) + (\overrightarrow{AC},\overrightarrow{AB}) + (\overrightarrow{FE},\overrightarrow{FB}) + 2k\pi,\ k\in\mathbb{Z}$$

$$= -\frac{\pi}{2} + \pi - \frac{\pi}{4} - \frac{\pi}{3} + \frac{\pi}{4} + 2k\pi,\ k\in\mathbb{Z}$$

$$= \frac{\pi}{6} + 2k\pi,\ k\in\mathbb{Z}$$